OCR

Friday 16 May 2014 – Afternoon

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Find the determinant of the matrix $\begin{pmatrix} a & 4 & -1 \\ 3 & a & 2 \\ a & 1 & 1 \end{pmatrix}$. **[3]**

2 The complex number 7 + 3i is denoted by z. Find

(i) $|z|$ and arg z, **[2]**

(ii) $\frac{z}{4-\mathrm{i}}$, showing clearly how you obtain your answer. **[3]**

3 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 3 \end{pmatrix}$ and **I** is the 2 × 2 identity matrix. Find

(i) $4\mathbf{A} - \mathbf{B} + 2\mathbf{I}$, **[2]**

(ii) $\mathbf{A}^{-1}$, **[2]**

(iii) $(\mathbf{AB}^{-1})^{-1}$. **[3]**

4 **(a)** Find the matrix that represents a shear with the y-axis invariant, the image of the point (1, 0) being the point (1, 4). **[2]**

(b) The matrix **X** is given by $\mathbf{X} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$.

(i) Describe fully the geometrical transformation represented by **X**. **[2]**

(ii) Find the value of the determinant of **X** and describe briefly how this value relates to the transformation represented by **X**. **[2]**

5 The cubic equation $2x^3 + 3x + 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = u + 2$ to find a cubic equation in u. **[3]**

(ii) Hence find the value of $\frac{1}{\alpha-2} + \frac{1}{\beta-2} + \frac{1}{\gamma-2}$. **[4]**

6 **(i)** Show that $\frac{1}{r^2} - \frac{1}{(r+2)^2} \equiv \frac{4(r+1)}{r^2(r+2)^2}$. **[2]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=1}^{n} \frac{4(r+1)}{r^2(r+2)^2}$. **[6]**

(iii) Find $\sum_{r=5}^{\infty} \frac{4(r+1)}{r^2(r+2)^2}$, giving your answer in the form $\frac{p}{q}$ where p and q are integers. **[2]**

7 The loci C_1 and C_2 are given by $\arg(z-2-2\mathrm{i}) = \frac{1}{4}\pi$ and $|z| = |z-10|$ respectively.

(i) Sketch on a single Argand diagram the loci C_1 and C_2. **[4]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$0 \leqslant \arg(z-2-2\mathrm{i}) \leqslant \tfrac{1}{4}\pi \text{ and } |z| \geqslant |z-10|.$$ **[3]**

8 **(i)** Show that $\sum_{r=n}^{2n} r^3 = \frac{3}{4}n^2(n+1)(5n+1)$. **[4]**

(ii) Hence find $\sum_{r=n}^{2n} r(r^2-2)$, giving your answer in a fully factorised form. **[5]**

9 The roots of the equation $x^3 - kx^2 - 2 = 0$ are α, β and γ, where α is real and β and γ are complex.

(i) Show that $k = \alpha - \frac{2}{\alpha^2}$. **[2]**

(ii) Given that $\beta = u + \mathrm{i}v$, where u and v are real, find u in terms of α. **[4]**

(iii) Find v^2 in terms of α. **[4]**

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5^n + 2^{n-1}$.

(i) Find u_1, u_2 and u_3. **[2]**

(ii) Hence suggest a positive integer, other than 1, which divides exactly into every term of the sequence. **[1]**

(iii) By considering $u_{n+1} + u_n$, prove by induction that your suggestion in part **(ii)** is correct. **[5]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$2a^2 + 6a - 15$	M1 M1 A1 **[3]**	Show correct expansion process for 3 × 3 Correct evaluation of any 2 × 2 Obtain correct answer i.s.w.	Condone sign errors for first M1 M2 for the "diagonal" method Det = $1/(2a^2 + 6a - 15)$ **only** A0
2	(i)		$\lvert z\rvert = \sqrt{58}$ or 7.62 $\arg z = 23.2(°)$ or 0.405 or 0.129π	B1 B1 **[2]**	Obtain correct value, 3 s.f. or better Obtain correct value, 3 s.f. or better	**arctan(3/7) gets B0**
	(ii)		*Either* $\frac{25}{17} + \frac{19}{17}$i *Or* $\frac{25}{17}$ and $\frac{19}{17}$	M1 A1 A1 **[3]** M1 A1 A1	Multiply numerator & denominator by conjugate Obtain correct numerator or real part Obtain correct denominator or imaginary part Find and attempt to solve a pair of simultaneous equations for real and imaginary parts of answer Obtain correct answers	$\frac{28+19i-3}{16+1}$ gets A0 A0
$\frac{1}{17}(25+19i)$**.3**	(i)		$\begin{pmatrix} 7 & 3 \\ -18 & 19 \end{pmatrix}$	B1 B1 **[2]**	2 elements correct All elements correct	Condone missing brackets in (i) (ii) & (iii).
	(ii)		$\frac{1}{14}\begin{pmatrix} 5 & -1 \\ 4 & 2 \end{pmatrix}$ or equivalent	B1 B1 **[2]**	Both diagonals correct, ignore determinant	$\frac{\begin{pmatrix} 5 & -1 \\ 4 & 2 \end{pmatrix}}{14}$ is OK for 2nd B1
$\frac{1}{14}\begin{pmatrix} 5 & -1 \\ 4 & 2 \end{pmatrix}$	(iii)		$(\mathbf{AB}^{-1})^{-1} = \mathbf{BA}^{-1}$ or $\mathbf{B}^{-1} = \frac{1}{7}\begin{pmatrix} 3 & -1 \\ -2 & 3 \end{pmatrix}$ $\frac{1}{14}\begin{pmatrix} 19 & -1 \\ 22 & 4 \end{pmatrix}$	B1 M1 A1 **[3]**	Correct result seen or used Multiplication attempt for any pair of 2 × 2 matrices, 2 elements correct, but not **I** Correct answer a.e.f.	

Question			Answer	Marks	Guidance	
4	(a)		$\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$	B1 B1 **[2]**	Each column correct	
	(b)	(i)		B1 B1 **[2]**	Rotation , 45º or π/4 clockwise or equivalent	Must be rotation and no other transformation, otherwise 0/2
	(b)	(ii)	(det **X** =) 1	B1 B1ft **[2]**	Correct value Scale factor for **area** or equivalent	 e.g. area unchanged
5	(i)		$2u^3 + 12u^2 + 27u + 25 = 0$	M1 A2 **[3]**	Substitute and attempt to simplify Obtain correct equation, A1 for only 1 error	 Missing = 0 is an error
	(ii)		*Either* $\frac{\sum \alpha'\beta'}{\alpha'\beta'\gamma'}$ $-\frac{27}{25}$ *Or* $25y^3 + 27y^2 + 12y + 2 = 0$ *Or* $\frac{\sum \alpha'\beta'}{\alpha'\beta'\gamma'}$	M1 A1 M1 A1ft **[4]** M1 A1ft M1 A1ft M1 A1 M1 A1	Combine 3 terms with correct denominator Obtain correct expression in their notation Attempt to use values from **(i)** correctly Obtain correct answer **with no errors seen** $y = \frac{1}{u}$ Obtain correct cubic equation, from their (i) Use correct symmetric function Obtain correct answer Combine 3 terms with correct denominator Obtain correct expression in their notation Expand numerator and denominator and use values from original equation correctly Obtain correct answer **with no errors seen**	 Must be ±c/**a and** ±d/**a** for M1 ft for their answer in (i) Condone ±, but must be "/2"

Question			Answer	Marks	Guidance	
6	(i)			M1 A1 **[2]**	Combine with a correct denominator Obtain **given** answer correctly	
	(ii)		$1+\frac{1}{4}-\frac{1}{(n+1)^2}-\frac{1}{(n+2)^2}$	M1 M1 A1 A1 M1 A1 **[6]**	Express as differences using **(i)** Attempt this for at least first 3 terms First 3 terms all correct Last 2 terms all correct Show correct cancelling Obtain correct answer i.s.w.	Final answer must be in terms of n
	(iii)		$\frac{61}{900}$	M1 A1 **[2]**	Start differences at n = 5 or S_∞ - S_4 Obtain correct answer, **with no errors seen**	$1+\frac{1}{4}-(1-\frac{1}{4}-\frac{1}{5^2}-\frac{1}{6^2})$
7	(i)			B1 B1 B1 B1 **[4]**	Half line, starting at (2, 2) with +ve slope upwards Vertical line Clearly x = 5 (must be vertical)	Not line segment e.g (0, 0) to (2, 2) Must be half line
	(ii)			B1 B1 B1 **[3]**	Shade below sloping line and above horizontal through their (2, 2) To right of their vertical line Completely correct diagram	Could be for a line segment, could be earned if C_2 horizontal 6/6 must be earned so far
8	(i)			M1 A1 M1 A1 **[4]**	Difference of sum to $2n$ and $n-1$ Correct unsimplified answer Sensible attempt to factorise, at least factor n^2 Obtain **given** answer no errors seen	$\frac{1}{4}(2n)^2(2n+1)^2-\frac{1}{4}(n-1)^2n^2$

Question			Answer	Marks	Guidance	
	(ii)			M1	Difference of **(i)** and another standard result	
				M1	Difference of $\sum_1^{2n} r - \sum_1^k r$ for $k = n-1$ or n	$\frac{1}{2}(2n)(2n+1) - \frac{1}{2}n(n-1)$
				A1	Obtain complete unsimplified expression	(i) - 2 x above
				M1	Sensible attempt to factorise, at least factor $n(n+1)$	
			¾ $n(n+1)^2(5n-4)$	A1	Obtain correct answer	$(n+1)(n+1)$ is OK for $(n+1)^2$
				[5]		
9	**(i)**		*Either*	M1	Substitute α into equation and rearrange	$\alpha^3 - k\alpha^2 - 2 = 0$
			$k = \alpha - \frac{2}{\alpha^2}$	A1	Obtain **given** answer a.e.f.	
				[2]		
			Or	M1	Substitute for k and x in terms of α and simplify	
				A1	Show simplification leads to consistency	e.g. "LHS = 0"
			Or	M1	Eliminate β and γ from symmetric functions	Don't penalise sign errors
				A1	Obtain **given** answer correctly	
	(ii)			B1	State or use $(\gamma) = u - iv$	
				M1*	Use sum of roots = (±)k (Can use ∑αβ with αβγ)	$\alpha + u + iv + u - iv = -(-k)$
				DM1	Rearrange to get u	$\alpha + 2u = \alpha - \frac{2}{\alpha^2}$
			$u = -\frac{2}{2\alpha^2}$ or better	A1	Obtain correct answer	
				[4]		

Question			Answer	Marks	Guidance	
	(iii)		$\alpha(u^2+v^2)=2$ or $2\alpha u+u^2+v^2=0$ $v^2=\frac{2}{\alpha}-\frac{1}{\alpha^4}$	M1* A1 DM1 A1 **[4]**	Use product of roots = (±)2 or $\sum\alpha\beta=0$ Obtain correct answer Substitute for u and rearrange to get v^2 Obtain correct answer a.e.f.	In terms of u and v
10	**(i)**		6 27 129	B1 B1 **[2]**	Obtain correct values Obtain 3rd correct value	
	(ii)		3	B1ft **[1]**	State a correct value	
	(iii)		$5^{n+1}+2^n$	B1 M1 A1 A1 B1 **[5]**	Correct expression for u_{n+1} seen Attempt to factorise $u_{n+1}+u_n$ Obtain correct simplified answer Clear explanation why u_{n+1} is divisible by 3 Clear statement of induction process	Any letter, usually k or n Must deal with powers of 5 and 2 Not $u_{n+1}+u_n$ divisible by 3 Provided other 4 marks earned